Effect of Wall Temperature on a Supersonic Turbulent Boundary Layer

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Measurement of mean flow profiles in a fully developed Mach 3 turbulent boundary layer with negligible pressure gradient is reported. Data were acquired at several streamwise locations for wall-to-total temperature ratios of 0.94, 0.71, and 0.54. The results demonstrate that the velocity defect formulation of the law-of-the-wake, which successfully correlates compressible, adiabatic boundary layers, is also valid for nonadiabatic flows. It is also shown that for adiabatic walls, the linear Crocco relation between total temperature and velocity does not provide a valid test of the nature of the boundary-layer flow for practical cases where the Prandtl number departs from unity. Finally, the turbulent shear stress, mixing length, and eddy viscosity were extracted from the "time-averaged" conservation equations using the measured mean flow profiles and found to be insensitive to wall temperature. In particular, the latter properties are in good agreement with earlier compressible, adiabatic correlations of turbulent transport properties.

Introduction

EXPERIMENTAL studies of turbulent supersonic boundary layers have been carried out to identify the effects of a variety of parameters including Mach number, freestream Reynolds number, heat transfer, and pressure gradient. The ultimate goal of such studies is to develop information leading to a better understanding of the structure of turbulent shear flows. In the absence of direct measurements of the fluctuating flow properties, the turbulent transport terms can be extracted from the time-averaged boundary-layer equations using measured mean flow profiles. The results form the basis for establishing analytic models of the turbulent transport processes. The latter then can be incorporated in numerical calculating schemes used for the computation of turbulent compressible flowfields. Turbulence modeling, however, has been constrained by the scarcity of available data and, consequently, flowfield predictions are possible over only a limited range of conditions.

This paper describes the results of a study of the effects of wall temperature on the turbulence structure of a Mach 3 boundary layer. Complete details on the results of this investigation are documented in Ref. 1, while the major findings are summarized here. Similar studies on the effect of heat transfer have been documented by Lee and Smith[2] at Mach 5 and by Watson[3] at Mach 20. However, the results of the former, carried out on the wall of a wind-tunnel channel similar to that used here, were concluded by the authors to have been influenced by upstream heat transfer. On the other hand, the latter tests, which utilized a flat-plate model, were directed primarily toward identifying low Reynolds number effects. The present experiments were conducted in the absence of these effects and therefore are considered representative of the fully equilibrated, zero pressure gradient boundary layer.

Two specific issues concerning boundary-layer structure have been addressed. First, it is shown that for an adiabatic wall, the Crocco energy relation does not provide a valid assessment of the quality of the boundary-layer flow in the case of nonunity Prandtl number. This confirms the analytical findings presented recently by Whitfield and High[4] and by Meier et al.[5] Second, using the Van Driest[6] velocity transformation, it was possible to correlate the measured mean flow boundary-layer profiles with Coles "Law-of-the-Wake"[7] for wall temperature ratios T_w/T_{0e} ranging from 1.0 to 0.54. In addition, the wall temperature was shown to have only a small effect on the turbulent transport properties, which were found to be in good agreement with the adiabatic correlations developed earlier by Maise and McDonald[8] over the Mach number range from 0 to 5.

Experiment

The measurements were carried out in the boundary layer over a flat plate installed with its upper surface in the centerplane of the Aeronutronic Mach 3 two-dimensional wind tunnel. The plate spanned the width of the tunnel with its sharp leading edge located 10-cm upstream of, and its trailing edge 55.9-cm downstream of the nozzle throat. Surface temperature was controlled by circulating liquid nitrogen through cooling coils attached to the underside of the plate. Static pressure ports and flush-mounted thermocouples were used to monitor surface conditions. The latter indicated that a relatively uniform plate temperature was established approximately 20-cm downstream of the throat, or about 20 boundary-layer thicknesses upstream of the first survey region, and that spatial variations in the plate temperature within the test section did not exceed 10°C.

An actuator extending through the upper wall of the tunnel mounted either a pitot pressure tube or a calibrated total temperature probe. The tip of the pitot probe consisted of a 0.010-cm-diam tube which tapered gradually to a 0.005-cm opening. Various probe corrections were found to be negligible, but interference effects caused by the proximity of the wall became evident when $y<0.015$ cm. This corresponds to about one third of the sublayer thickness. The total temperature probe consisted of an unshielded thermocouple with its 0.004-cm-diam wires welded to form a 0.013-cm-diam sphere.

Tests were carried out at $M_\infty=3$ for a tunnel stagnation pressure of 0.97×10^5 N/m^2 and a stagnation temperature of 318 K, corresponding to a freestream Reynolds number of 6.57×10^6m^{-1}. The momentum Reynolds number Re_θ varied from 3200 to 4400. Measurements were made for wall temperatures $T_w/T_{0e}=0.94$ (adiabatic case), 0.71, and 0.54, and at each wall condition detailed mean flow surveys were performed at streamwise stations 35, 38.8, and 42.5 cm from

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Table 1 Boundary-layer properties

x sta, cm	δ, cm	δ^*, cm	θ, cm	Re_θ	C_f[a]
		$T_w/T_{0e}=0.94$			
35	0.846	0.274	0.0508	3273	0.00199
38.8	0.899	0.292	0.0533	3487	0.00196
42.5	0.960	0.309	0.0584	3816	0.00192
		$T_w/T_{0e}=0.714$			
35	0.820	0.244	0.0518	3384	0.00216
38.8	0.856	0.261	0.0536	3507	0.00215
42.5	0.899	0.276	0.0584	3862	0.00210
		$T_w/T_{0e}=0.54$			
35	0.826	0.223	0.0538	3509	0.00233
38.8	0.861	0.263	0.0612	3964	0.00227
42.5	0.876	0.250	0.0632	4203	0.00224

[a] Calculated from skin friction correlation.[9]

the nozzle throat. Prior to the final measurements, extensive surveys were performed to insure the two dimensionality and repeatability of the boundary layer. The local freestream static pressure was determined from the measured local freestream pitot pressure and the tunnel stagnation pressure. These pressures, which were assumed constant across the boundary layer, indicated a slight adverse streamwise pressure gradient corresponding to a nominal value of $\beta \equiv (\theta/\tau_w)\mathrm{d}p/\mathrm{d}x = 0.04$. Subsequent analysis of the data, however, showed that this gradient was too small to cause an observable departure from a zero pressure gradient flow.

Results

Mean Flow Profiles

Mean flow properties were calculated from the pitot pressure and total temperature measurements by means of standard gasdynamic equations using an iterative procedure to account for the calibrated recovery factor characteristics of the T_0 probe. All calculations were referenced to the position of the pitot probe. Since this differed from the location of the T_0 probe, the recovery temperature at the pitot probe position was determined from a simple three-point interpolation scheme. The pitot pressure variation in the freestream was observed to be uniform or showed a slight linear variation with increasing y. The boundary-layer thickness δ was defined as that point where the pitot pressure profile within the layer merged with the straight line representing the distribution of pitot pressure in the freestream. On this basis, δ ranged from approximately 0.8 to 1.0 cm, depending on x and T_w. Table 1 summarizes the calculated integral properties of the boundary layer as a function of x position and wall temperature and includes the skin-friction coefficient calculated in the next section.

The velocity profiles, normalized by the edge value of u and the boundary-layer thickness δ, were found to be independent of streamwise location. Furthermore, as shown in Fig. 1, where u/u_e vs y/δ at the midsurvey station has been plotted for each wall temperature condition, the normalized velocity profiles are also independent of heat transfer. In view of the short intervals between survey stations, the agreement between profiles at a given wall condition was not a conclusive test of similarity nor that the turbulence was fully developed. However, the results of data analysis described later confirm the maturity of the boundary-layer flow.

For a given wall temperature, the normalized total temperature profiles were also found to be independent of streamwise location. The effect of heat transfer on the total temperature profile is indicated on Fig. 2 where T_0/T_{0e} vs y/δ at the midsurvey station has been plotted for each wall temperature. The data show clearly the T_0 overshoot expected for nonunity Prandtl number flows and indicate that the thickness of the thermal boundary layer is similar to the

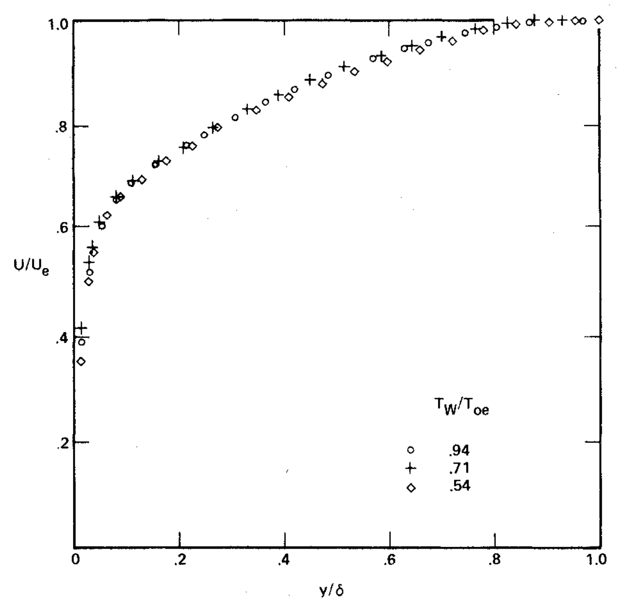

Fig. 1 Nondimensional velocity profiles showing the effect of wall temperature.

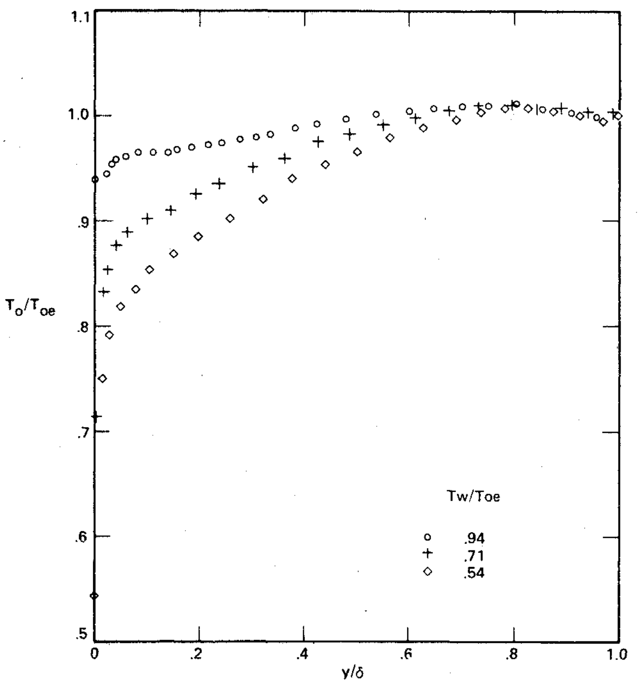

Fig. 2 Nondimensional total temperature profiles showing the effect of wall temperature.

values of δ defined on the basis of the pitot pressure measurements. The T_0 profiles indicate that the sublayer thickness is about 0.05 y/δ and that, especially for the nonadiabatic cases, approximately one half the change in T_0 occurs across the viscous sublayer.

Skin Friction

It was originally intended to measure skin friction using a Preston tube and, provided that the sublayer could be probed in sufficient detail, from the slope of the experimentally determined velocity profile. However, the velocity profile approached, but did not attain, a linear variation near the wall, so that this method could provide only a lower bound to the skin-friction coefficient. In addition, the Preston tube measurements indicated C_f values considerably higher than implied by the trend of the velocity data. This is apparently a consequence of the large scatter inherent in existing Preston tube correlations.

Consequently, the skin-friction coefficient was calculated using two approaches. The first involved the skin-friction correlation of Hopkins and Inouye,[9] which is based on the Karman-Schoenberr equation and the Van Driest velocity transformation, and the second was based on Coles "Law-of-the-Wake" velocity correlation:[7]

$$u^+ = u^*/u_\tau = 2.43\ \ell n(y^+) + 5 + 2.43\bar{\pi} W(y/\delta) \tag{1}$$

In the foregoing expression, u^* is the Van Driest generalized velocity, $y^+ = yu_\tau/\nu_w$, u_τ is the friction velocity, $\bar{\pi}$ is a parameter representing the strength of the wake component of the boundary layer, and W is Coles tabulated wake function which can be approximated by $2\sin^2(\pi y/2\delta)$. The quantities u_τ, δ, and $\bar{\pi}$ are treated as unknown constants and are determined by adjusting their values until the velocity data fit Eq. (1) with a minimum rms error.

The skin-friction coefficients calculated from the method of Hopkins and Inouye[9] varied from 0.0019 to 0.0023 depending on x and T_w (see Table 1) and were consistently 10% greater than those deduced from the Law-of-the-Wake correlation. However, the latter also resulted in values of δ about 10% smaller than those defined from the pitot pressure profiles. This is because the velocity approaches its freestream value slightly faster than the other properties (density, total temperature, etc.). Since the Coles correlation[7] relies only on the transformed velocity, the equivalent boundary-layer thickness can be expected to be less than δ based on the experimental observations. In fact, the value of δ found in this manner can be considered as the velocity boundary-layer thickness. Consequently, the Law-of-the-Wake correlation procedure was modified by specifying δ as its known value derived from the pitot pressure profile, and the curve fit process repeated to determine only u_τ and $\bar{\pi}$. The skin-friction coefficients found by the modified procedure were slightly larger than the results of the original method but still were a few percent less than the results of Hopkins and Inouye.[9] It was noted that with both procedures, the value of the wake parameter $\bar{\pi}$ ranged from 0.5 to 0.8 which is close to the value of 0.6 that is characteristic of flat-plate, zero pressure gradient, fully developed boundary layers.

Correlation of Velocity Profiles

Written in defect form, Eq. (1) becomes

$$u_e^+ - u^+ = -2.43\ell n(y/\delta) + 1.46(2-W) \tag{2}$$

where $\bar{\pi}$ has been set equal to 0.6, its zero pressure gradient, flat-plate value. Equation (2) is plotted as the solid line in Fig. 3, which includes also the data for the nine survey conditions (three survey stations and three wall temperatures) of the present tests. The calculations for Fig. 3 were based on values of δ derived from the pitot pressure profiles and skin-friction coefficients determined from the Hopkins and Inouye[9] correlation. It is noted that all of the measured profiles are in good agreement with the correlation curve. This implies, therefore, that at each wall temperature, the boundary layer is fully developed and free from nonequilibrium or relaxation effects. Furthermore, Eq. (2) is seen to be equally valid for the three wall temperature levels tested. This is in contrast to the earlier findings of Maise and McDonald,[8] who examined six different nonadiabatic velocity profiles by two different investigators[10,11] and found poor agreement with the correlation curve described by Eq. (2). Consequently, they restricted their subsequent development of a correlation for eddy viscosity to adiabatic flows. As shown later, the results of the present tests remove this restriction. More recently, Owen and Horstman[12] reported good agreement between Eq. (2) and their cold wall measurements at Mach 7.2 and $T_w/T_{0e} = 0.46$. However, the present study yields the first

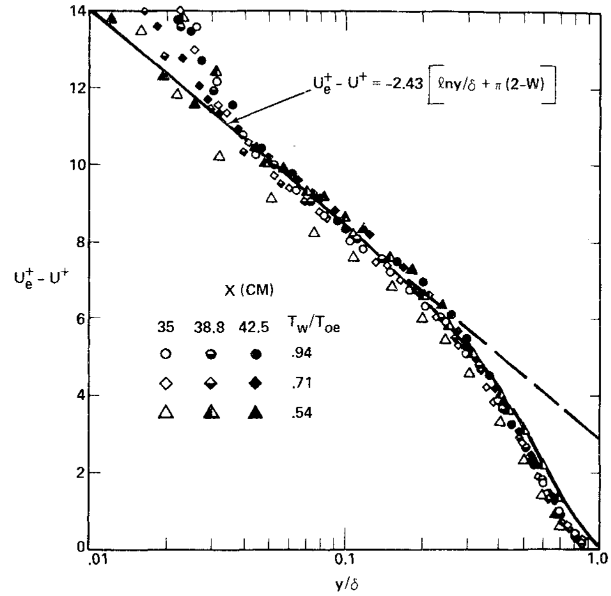

Fig. 3 Law-of-the-Wake correlation in velocity defect coordinates.

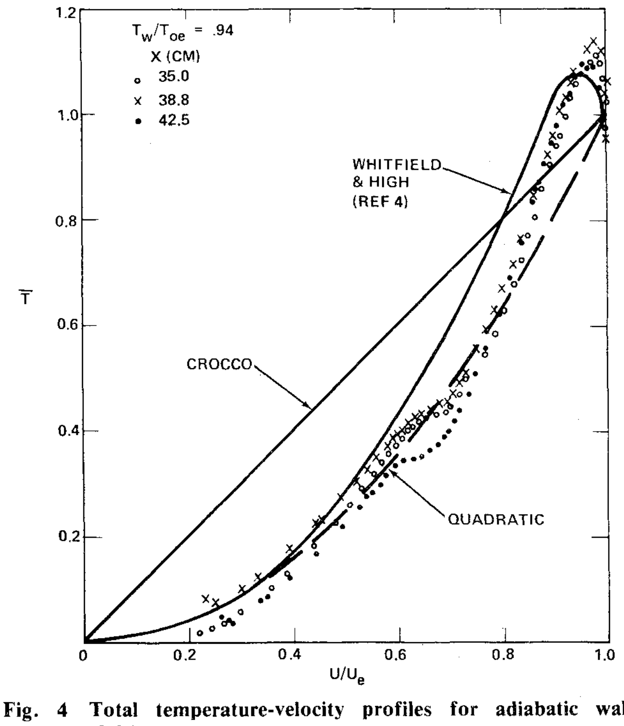

Fig. 4 Total temperature-velocity profiles for adiabatic wall, $T_w/T_{0e} = 0.94$.

systematic evidence that the Law-of-the-Wake described by Eq. (2) provides a valid correlation of compressible flows with heat transfer.

It should be noted that some improvement in the agreement between the data and Eq. (2) could be obtained using a smaller δ. This was done, e.g., by Owen and Horstman[12] who redefined their original δ by using a power law to represent their velocity profile. This new δ, used in their velocity defect correlation, was 13% smaller than that based on the pitot pressure profile. In the present case, the curve fit to Eq. (1) provides smaller values of δ but this implies different values for C_f and $\bar{\pi}$, which would also influence the comparison with Eq. (2). Furthermore, changes in $\bar{\pi}$ would depart from the assumption common to all the profiles considered by Maise and McDonald.[8] This serves to illustrate the difficulties in defining boundary-layer thickness in compressible flows, but does not detract from the success of Eq. (2) in correlating the experimental profiles.

Total Temperature Velocity Profiles

It has become common practice to compare experimental profiles to the linear Crocco relation $\bar{T}=u/u_e$. The Crocco relation, however, is restricted to unity Prandtl number, an assumption made to eliminate the turbulent shear stress terms from the energy equation. As a result, the simple Crocco expression cannot predict the total temperature overshoot in adiabatic, nonunity Prandtl number flows that have been observed in numerous experimental investigations and confirmed by the numerical analysis of Van Driest.[13] It has been further observed experimentally that the $\bar{T}-u/u_e$ profiles for boundary layers growing over aerodynamic models (flat plates, cones) differs from those on nozzle walls (see, for example, the data review in Ref. 14 as well as the results of Refs. 15 and 16). The latter conform more closely to the Walz[17] quadratic law $\bar{T}=(u/u_e)^2$ which is valid for $P_r<1.0$ but is limited to zero heat transfer. However, the Walz relation also ignores the turbulent shear stress terms in the energy equation and, similar to the Crocco expression, is unable to account for the T_0 overshoot in the adiabatic case. The quadratic behavior has been attributed by several investigators (see e.g., Refs. 18 and 19) to various effects associated with the upstream history of the nozzle expansion which tend to delay the attainment of an equilibrium boundary-layer structure.

More recently, Meier et al.,[5] and Whitfield and High[4] have examined the $\bar{T}-u/u_e$ relationship for nonunity Prandtl number flows by including approximate models for the turbulent shear stress distribution across the boundary layer. The former present a numerical analysis, while the latter derive an analytic solution and, in both cases, their results have been substantiated by comparison with experimental data. In the present case, the data from the three survey stations for the adiabatic case are plotted in Fig. 4, which includes the Crocco relation, the quadratic expression, and the results of the Whitfield and High[4] theory. It is immediately clear that the linear Crocco relation is a completely inadequate representation of the data, which instead are in good accord with both the prediction of Whitefield and High[4] and the trend of the quadratic law. More important, the T_0 overshoot calculated from the theory confirms the experimental observations. The "kink" in the profile at $u/u_e=0.6$ corresponds to the edge of the sublayer and is similar to the behavior observed by other investigators.[15,16,20]

The results for the cold wall cases are shown in Figs. 5 and 6 for $T_w/T_{0e}=0.71$ and 0.54, respectively, where, again, both figures include the Crocco relation and the data for the three survey stations. It is immediately evident that with heat transfer, even at moderate rates, the $\bar{T}-u/u_e$ profile becomes nearly linear and that the Crocco relation is now an accurate approximation of the data. In fact, the theory of Whitfield and High[4] indicates that the shift to a linear-like behavior occurs quickly as the wall temperature is decreased below its adiabatic value.

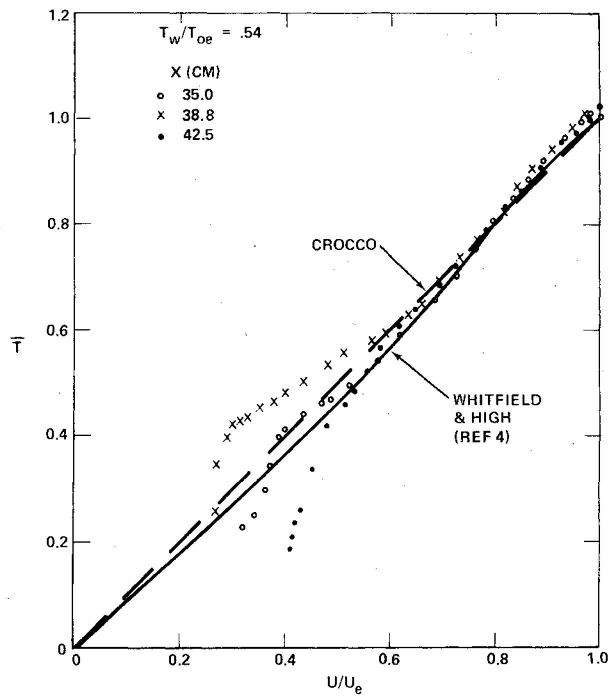

Fig. 6 Total temperature-velocity profiles for $T_w/T_{0e}=0.54$.

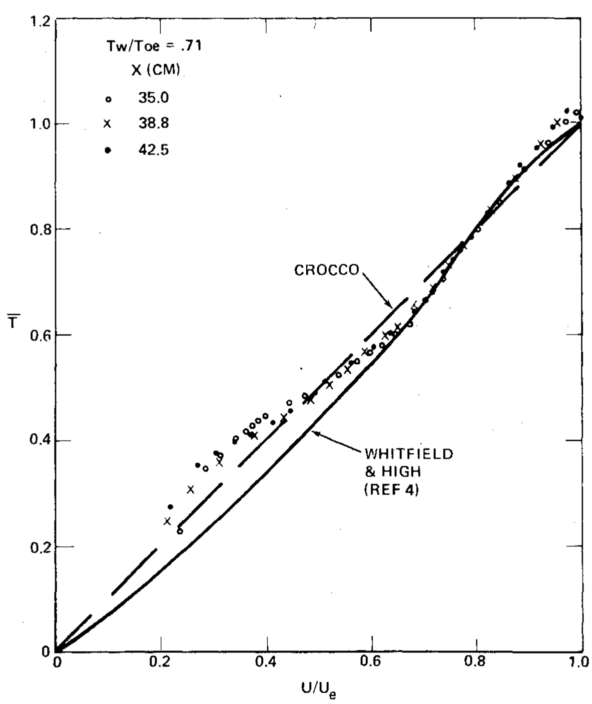

Fig. 5 Total temperature-velocity profiles for $T_w/T_{0e}=0.71$.

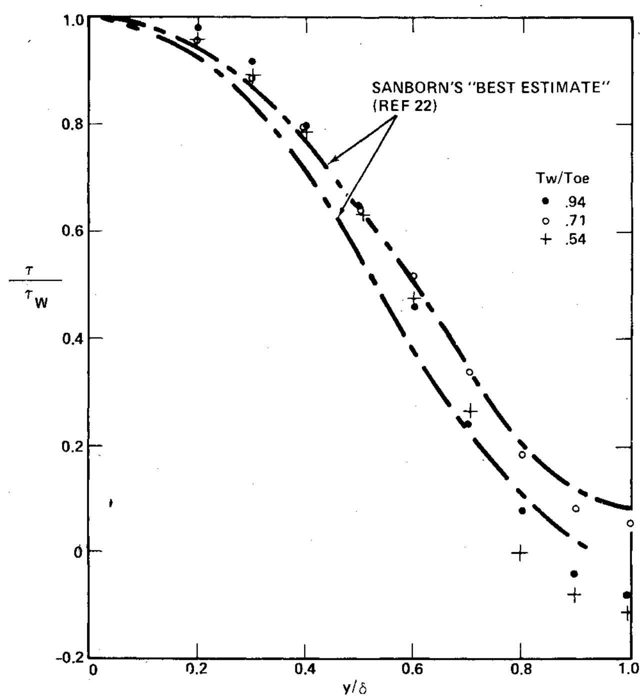

Fig. 7 Turbulent shear stress distributions.

The present results show that the $\bar{T}$ vs u/u_e profiles in the nonunity Prandtl number boundary-layer flows can be accurately calculated provided that the turbulent shear stresses are taken into account. While the Crocco relation appears to be valid for finite heat-transfer rates, it is not an appropriate basis of comparison for the case of an adiabatic wall where $P_r < 1.0$. In view of the agreement between the data and the Whitfield and High theory, it is suggested that relaxation effects in nozzle wall boundary layers may be less important then previously considered.

There are a number of boundary-layer transformations and correlations which utilize the Crocco relation in their derivation. In view of the preceding finding, the validity of these correlations may be questioned. It should be pointed out, however, that in the adiabatic case, the Crocco parameter $\bar{T}$ is extremely sensitive to uncertainties in the total temperature. In the present instance, for example, where $T_{0e} - T_w$ is only 6-7% of T_{0e}, the extremes of a constant T_0 on the one hand, and a linear T_0 variation on the other, produce a difference of only a few percent in the corresponding profiles of velocity and density. Hence, the mathematical simplification afforded by the Crocco relation justifies its use in approximating the $T_0 - u$ variation. The point to be emphasized is that the Crocco relation, cast in its usual form $\bar{T}$ vs u/u_e, cannot be used to judge the quality of the boundary-layer flow.

Turbulent Transport Properties

Under suitable assumptions, the turbulent shear stress can be extracted from experimental mean flow data. The time-averaged conservation equations for the two-dimensional, flat-plate boundary layer are

Continuity

$$\partial(\rho u)/\partial x + \partial(\rho\tilde{v})/\partial y = 0 \tag{3}$$

Momentum

$$\partial(\rho u^2)/\partial x + \partial(\rho u\tilde{v})/\partial y = -\mathrm{d}p/\mathrm{d}x + \partial\tau/\partial y \tag{4}$$

where $\tilde{v} = v + \overline{\rho' v'}/\rho$ and the total shear stress τ is the sum of a laminar and a turbulent term:

$$\tau = \tau_L + \tau_T = \mu\partial u/\partial y - \overline{(\rho v)'u'}$$

The turbulent shear stress term can be expanded as follows:

$$\tau_T = -\overline{(\rho v)'u'} = -\rho\overline{(u'v')} - v\overline{(\rho'u')} - \overline{(\rho'u'v')}$$

If the last two terms on the right-hand side are assumed to be negligible, the expression for the turbulent shear reduces to the Reynolds shear stress of incompressible flow, namely $-\rho\overline{(u'v')}$. However, for complete generality, the compressible formulation should be retained.

In the preceding paragraphs, the present data were shown to be representative of a fully developed turbulent boundary layer, and evidence was presented which indicates that the mean flow profiles, i.e., u/u_e, ρ/ρ_e, etc., are functions of only y/δ. This allows then $\partial f/\partial x$ to be replaced with $(f/f_e)\mathrm{d}f_e/\mathrm{d}x$ for constant y/δ, where $f = \rho u$ or ρu^2. Integrating Eq. (3) with respect to y yields an expression for $\rho\tilde{v}$ which can be substituted into Eq. (4). The latter can then be integrated with respect to y/δ yielding the following expression for the total shear[21]:

$$\frac{\tau}{\rho_e u_e^2} = \frac{C_f}{2} + \left[\frac{\delta}{\rho_e u_e^2}\frac{\mathrm{d}}{\mathrm{d}x}(\rho_e u_e^2) + \frac{\mathrm{d}\delta}{\mathrm{d}x}\right]\left[\int_0^{y/\delta}\frac{\rho}{\rho_e}\left(\frac{u}{u_e}\right)^2\mathrm{d}\left(\frac{y}{\delta}\right)\right]$$
$$-\left[\frac{\delta}{\rho_e u_e}\frac{\mathrm{d}}{\mathrm{d}x}(\rho_e u_e) + \frac{\mathrm{d}\delta}{\mathrm{d}x}\right]$$
$$\times\left[\frac{u}{u_e}\int_0^{y/\delta}\frac{\rho}{\rho_e}\frac{u}{u_e}\mathrm{d}\left(\frac{y}{\delta}\right)\right] + \frac{\delta}{\rho_e u_e^2}\frac{y}{\delta}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{5}$$

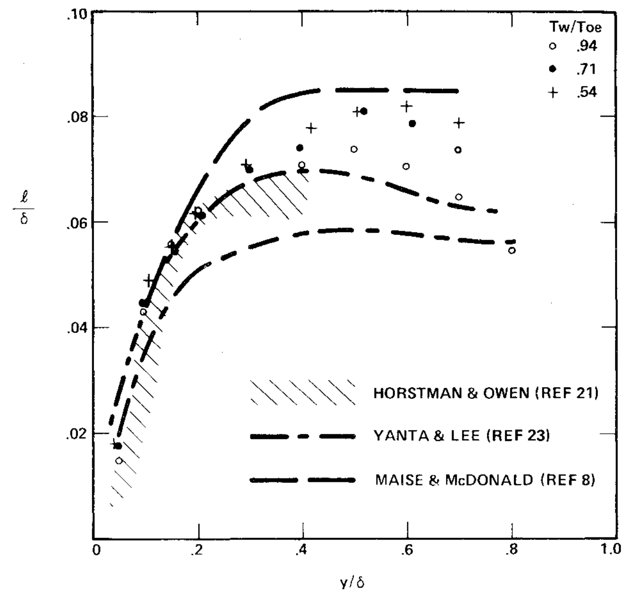

Fig. 8 Mixing length distributions.

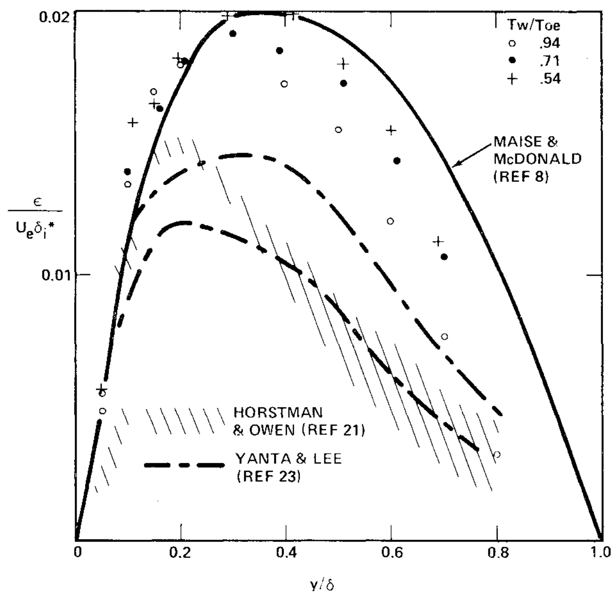

Fig. 9 Eddy viscosity distributions.

where it has been assumed that the static pressure is constant across the boundary layer.

Evaluation of the shear stress using Eq. (5) was restricted to the midsurvey station ($x = 38.8$ cm) for each wall temperature condition. The x derivatives of $\rho_e u_e$, $\rho_e u_e^2$, and p were found by applying the simple two-point differencing scheme to the edge properties, yielding average values between the first and second station and the second and third stations. These values were then further averaged to determine the derivatives at the midsurvey station. Since the boundary-layer thickness δ was determined from inspection of the measured pitot pressure profile, its value was influenced by the density of data points near the edge of the layer and by small errors in the pressure measurements. Therefore, δ lacks the precision of the integral properties, which depend on the variation of velocity and density within the boundary layer and are insensitive to the location of the edge of the layer. The uncertainties in δ are further accentuated in determining $\mathrm{d}\delta/\mathrm{d}x$, since the increase in the boundary-layer thickness is relatively small over the limited interstation intervals of the present tests. Consequently, it was assumed that δ could be related to the momentum thickness θ through a simple form factor k, so that $\delta = k\theta$ and $d\delta/dx = kd\theta/dx$. The value of k was the average value for the three survey stations.

For each wall temperature, a plot of the turbulent shear stress, normalized by wall shear, is shown as a function of y/δ in Fig. 7. In each case, it is seen that the shear stress does not completely vanish at $y/\delta = 1.0$, probably reflecting errors in evaluating the streamwise derivatives. However, the discrepancy is not large, with the maximum residue occurring for the cold wall case where it is negative in sign and only about 12% of the wall shear. The results, overall, are considered quite satisfactory. Figure 7 also includes the "best estimate" suggested by Sanborn[22] based on his review of both hot-wire measurements and mean flow calculations for Mach numbers ranging from 0 to 6.8. The present results are seen to be in good agreement with Sanborn's distribution and furthermore indicate that the normalized shear stresses are insensitive to wall temperature.

With the turbulent shear stress distribution determined from the mean flow profiles, it is possible to calculate the mixing length ℓ, and the eddy viscosity ϵ. These quantities are defined as:

$$\tau_T = \rho\ell^2 \,|\partial u/\partial y|\, \partial u/\partial y = \rho\epsilon \partial u/\partial y \tag{6}$$

From the preceding, the normalized mixing length can be expressed as:

$$\frac{\ell}{\delta} = \left[\frac{(\tau_T/\rho_e u_e^2)}{(\rho/\rho_e)[\partial(u/u_e)/\partial(y/\delta)]^2}\right]^{1/2} \tag{7}$$

The mixing length distribution is plotted in Fig. 8, where it is compared to the results of Horstman and Owen,[21] obtained at $M_\infty = 7.2$ and Re_θ from ranging 4900 to 9700, and to those of Yanta and Lee[23] acquired at $M_\infty = 2.9$ and Re_θ from 10,000 to 14,000. Included in the figure is the mixing length calculated by Maise and McDonald[8] on the basis of their earlier correlation of two-dimensional, adiabatic supersonic flow measurements. They found that for M_∞ up to 5 and Re_θ from 10^3 to 10^5, the Mach number and Reynolds number effects are quite small and generally within the experimental uncertainty of ℓ/δ, so that for this range of conditions the mixing distribution could be represented by the incompressible variation of ℓ/δ vs y/δ. As seen, the present results compare favorably with the Maise and McDonald[8] model throughout the entire boundary layer, although they indicate slighly lower values fo $y/\delta > 0.5$. The present data also show a slight separation due to T_w/T_{0e} and, while the separation is small, it suggests that ℓ/δ increases as T_w/T_{0e} decreases. It is of interest to note that the results of Horstman and Owen[21] and Yanta and Lee[23] are in agreement with the present findings and the correlation of Maise and McDonald[8] only near the wall ($y/\delta < 0.2$). For larger y/δ, the results of Refs. 21 and 23 are 20 to 30% less than the value indicated by the correlation.

Solving Eq. (6) for ϵ yields:

$$\frac{\epsilon}{u_e\delta_i^*} = \frac{\tau_T/\rho_e u_e^2}{(\rho/\rho_e)[\partial(u/u_e)/\partial(y/\delta](\delta_i^*/\delta)} \tag{8}$$

where it has been shown by Maise and McDonald[8] that when ϵ is normalized by the kinematic boundary-layer thickness δ_i^*, its distribution across the boundary layer is insensitive to Mach number and to Re_θ. The present calculations of eddy viscosity are plotted in Fig. 9, where they are compared to those of Horstman and Owen[21] and Yanta and Lee[23] and to the Maise and McDonald[8] model. It is seen that for $y/\delta > 0.2$ the present results are significantly larger than both those of Refs. 21 and 23. However, they are in reasonable agreement with the predictions of Maise and McDonald,[8] although yielding slighly larger values for $y/\delta < 0.3$ and slightly smaller values of normalized eddy viscosity for $y/\delta > 0.3$. In particular, the peak value of $\epsilon/u_e\delta_i^* = 0.02$ at $y/\delta \sim 0.3$ calculated in the present case is in agreement with the maximum value deduced from the Maise and McDonald[8] model. Again, the trend of the present data suggests a slight wall temperature effect with $\epsilon/u_e\delta_i^*$ increasing as T_w/T_{0e} is reduced.

Summary

The results of the present study provide systematic evidence that, for moderate heat-transfer rates, the structure of turbulent supersonic boundary layers is unaffected by heat transfer to the wall. For wall temperatures ranging from 0.94 to 0.54, the measured mean velocity profiles were successfully correlated by the Law-of-the-Wake expression and the turbulent transport properties deduced from the mean flow profiles were found to be in good agreement with previous results for compressible, adiabatic flows. It was also observed that with decreasing heat transfer, the experimental total temperature-velocity profiles showed significant departure from the linear Crocco relation. This confirms earlier theoretical findings which demonstrate that for the adiabatic case with nonunity Prandtl number, the Crocco relation is not characteristic of fully developed turbulent flow.

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